Euclidean Action and Spacetime thermodynamics

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It has long been indicated in the work of Gibbons and Hawking [1] that Euclidean action is related to thermodynamic variables, and many attempts have been made to derive Einstein's equations as thermodynamic identities [2, 3]. The motivation for these speculations is coming because free energy density is the Euclidean Lagrangian which is precisely the scalar curvature for the static spacetime(s), which states that minimizing the Einstein -Hilbert action can be equivalent thought of as minimizing the macroscopic free energy. We claim that it holds only in static spacetime; in general, the Euclidean action gives the additional extrinsic curvature-dependent terms that seem to violet this free energy connection to Einstein-Hilbert action, spoiling the idea of deriving Einstein's equations as thermodynamic identities in general spacetime. Since entropy is related to free energy as $\beta F = \beta U - TS$, it must be modified in the non-static case. In this paper, we propose a correct definition of Euclidean action for a wide range of spacetime by using our covariant Wick rotation [4] hence the correct definition for free energy for general curved spacetime. We also discuss the deep connection between other thermodynamic variables and geometric variables. In the last, we discuss how the modified Euclidean action corrects the black hole entropy formula for stationary spacetime by that gives the Bekenstein-Hawking entropy for static Killing horizons in four dimensions.

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