Relationship between the metric tensor and the field tensor.The structure of spacetime.

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Abstract

The document presents the main conclusions from the author's last article, regarding the structure of spacetime. Relationship between the metric tensor and field tensors are discussed, and possible implications for further research of the presented approach are analysed.

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I. MAIN CONCLUSIONS FROM THE RESEARCH

According to [1], stress-energy tensor $T^{\alpha\beta}$ for a system in a given spacetime described by a metric tensor $g^{\alpha\beta}$ may be defined as

$$T^{\alpha\beta} = \varrho\, U^{\alpha} U^{\beta} - \left(c^2 \varrho + \Lambda_{\rho}\right) \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \tag{1}$$

where ϱ_o is for rest mass density and

$$\varrho \equiv \varrho_o \gamma \tag{2}$$

$$\frac{1}{\xi} \equiv \frac{1}{4}\, g_{\mu\nu}\, h^{\mu\nu} \tag{3}$$

$$\Lambda_{\rho} \equiv \frac{1}{4\mu_o} \mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} g_{\alpha\beta} \tag{4}$$

$$h^{\alpha\beta} \equiv 2\, \frac{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}}{\sqrt{\mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma}\, g_{\mu\beta}\, \mathbb{F}_{\alpha\eta}\, g^{\eta\xi}\, \mathbb{F}^{\mu}{}_{\xi}}} \tag{5}$$

where $\mathbb{F}^{\alpha\beta}$ represents electromagnetic field tensor.

The stress–energy tensor for electromagnetic filed, denoted as $\Upsilon^{\alpha\beta}$ may be presented as follows

$$\Upsilon^{\alpha\beta} \equiv \Lambda_{\rho} \left(g^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) = \Lambda_{\rho} g^{\alpha\beta} - \frac{1}{\mu_o} \mathbb{F}^{\alpha\delta}\, g_{\delta\gamma}\, \mathbb{F}^{\beta\gamma} \tag{6}$$

The pressure p in the system is equal to

$$p \equiv c^2 \varrho + \Lambda_{\rho} \tag{7}$$

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so the stress-energy tensor $T^{\alpha\beta}$ for a system may be then denoted as just

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - \frac{p}{\Lambda_\rho}\Upsilon^{\alpha\beta} \tag{8}$$

It has also been shown that the above stress-energy tensor can be extended by other fields without losing the further presented properties of the solution.

The described solution and the properties of spacetime obtained thanks to it, require a slight correction to the continuum mechanics. Thanks to this amendment in flat Minkowski spacetime occurs

$$\partial_\alpha U^\alpha = -\frac{d\gamma}{dt} \quad \rightarrow \quad \partial_\alpha\, \varrho U^\alpha = 0 \tag{9}$$

thus denoting four-momentum density as $\varrho U^\mu = \varrho_o \gamma\, U^\mu$, total four-force density f^μ acting in the system is

$$f^\mu \equiv \varrho A^\mu = \partial_\alpha \varrho U^\mu U^\alpha \tag{10}$$

Denoting rest charge density in the system as ρ_o and

$$\rho \equiv \rho_o \gamma \tag{11}$$

electromagnetic four-current J^α is equal to

$$J^\alpha \equiv \rho\, U^\alpha = \rho_o \gamma\, U^\alpha \tag{12}$$

In the flat Minkowski spacetime, total four-force density f^α acting in the system calculated from $\partial_\beta\; T^{\alpha\beta} = 0$ is the sum of electromagnetic (f^α_{EM}), gravitational (f^α_{gr}) and other (f^α_{oth}) four-force densities

$$f^\alpha = \begin{cases} f^\alpha_{EM} \equiv -\Lambda_\rho\, \partial_\beta \xi\, h^{\alpha\beta} \quad (electromagnetic) \\ + \\ f^\alpha_{gr} \equiv \left(\eta^{\alpha\beta} - \xi\, h^{\alpha\beta}\right) \partial_\beta\, p \quad (gravitational) \\ + \\ f^\alpha_{oth} \equiv \frac{\varrho c^2}{\Lambda_\rho} f^\alpha_{EM} \quad (other) \end{cases} \tag{13}$$

As was shown in [1], in curved spacetime ($g_{\alpha\beta} = h_{\alpha\beta}$) part of the stress-energy tensor $T^{\alpha\beta}$ related to fields vanishes, and presented method reproduces Einstein Field Equations with an accuracy of $\frac{4\pi G}{c^4}$ constant and with cosmological constant Λ dependent on invariant of electromagnetic field tensor $\mathbb{F}^{\alpha\gamma}$

$$\Lambda = -\frac{\pi G}{c^4 \mu_o} \mathbb{F}^{\alpha\mu} h_{\mu\gamma} \mathbb{F}^{\beta\gamma} h_{\alpha\beta} = -\frac{4\pi G}{c^4} \Lambda_\rho \tag{14}$$

where $h_{\alpha\beta}$ appears to be metric tensor of the spacetime in which all motion occurs along geodesics and where Λ_ρ describes vacuum energy density.

It is also shown, that Einstein tensor describes the spacetime curvature related to vanishing in curved spacetime four-force densities $f^\alpha_{gr} + f^\alpha_{oth}$.

The presented solution creates a coherent picture in which spacetime is in fact a way of perceiving the field (in this case: electromagnetic field). This solution allows for further development, introducing additional fields, different parameterization and simple transformation between Minkowski spacetime and curvilinear reference systems.

It also shows, that description of motion in curved spacetime and its description in flat Minkowski spacetime with fields are equivalent. This allows for a significant simplification of research, because the results obtained in flat Minkowski spacetime can be easily transformed into curved spacetime. The last missing link seems to be the quantum description.

II. POTENTIAL, FARTHER CONSEQUENCES

It is discussed in [2], that by imposing additional condition on normalized stress-energy tensor in flat Minkowski spacetime with fields

$$0 = \partial_\beta \left(\frac{T^{\alpha\beta}}{\eta_{\mu\gamma} T^{\mu\gamma}} \right) + \partial^\alpha \ln \left(\eta_{\mu\gamma} T^{\mu\gamma} \right) \tag{15}$$

one obtains following results

- Lagrangian density for the systems appears to be equal to $\mathcal{L} = \Lambda_\rho = \frac{1}{4\mu_o} \mathbb{F}^{\alpha\beta} \mathbb{F}_{\alpha\beta}$
- Stress-energy tensor may be simplified to familiar form: $T^{\alpha\beta} = \frac{1}{\mu_o} \mathbb{F}^{\alpha\gamma} \partial^\beta \mathbb{A}_\gamma - \Lambda_\rho \eta^{\alpha\beta}$
- $H^\beta \equiv -\frac{1}{c} \int T^{0\beta} \, d^3x$ acts as cannonical four-momentum for the point-like particle, it includes electromagnetic four-potential and other terms responsible for other fields

- The vanishing four-divergence of the canonical four-momentum H^β turns out to be the consequence of Poynting theorem
- Some gauge of electromagnetic four-potential may be expressed as $\mathbb{A}^\mu = -\frac{\Lambda_\rho}{p}\frac{\varrho_o}{\rho_o}U^\mu$

One may express canonical four-momentum H^μ as

$$H^\mu = P^\mu + V^\mu \tag{16}$$

where P^μ is four-momentum and where V^μ is related to the field and is calculated in [2] as

$$V^\mu = q\mathbb{A}^\mu + \mathbb{S}^\mu + Y^\mu \tag{17}$$

where $\mathbb{A}^\mu$ is electromagnetic four-potential, $\mathbb{S}^\mu$ due to its properties, may be associated with some description of the spin

$$\mathbb{S}^\mu = -\int \frac{\epsilon_o \mathbb{A}^0}{\gamma} \mathbb{F}^{0\nu} \partial_\nu U^\mu \, d^3x \tag{18}$$

where ϵ_o is electric vacuum permittivity, and where Y^μ is related to Poyinting four-vector

$$Y^\mu = \frac{1}{c} \int \Upsilon^{0\mu} \, d^3x \tag{19}$$

If, indeed, in the absence of fields, Lagrangian, Hamiltonian and Action vanish...
Since in the limit of the inertial system one gets $P^\mu X_\mu = mc^2\tau$, therefore, to ensure vanishing Hamilton's principal function in the inertial system, one can expect that

$$V^\mu X_\mu \equiv -mc^2\tau \tag{20}$$

what yields vanishing in the inertial system Lagrangian in form of

$$-\gamma L = F^\mu X_\mu \tag{21}$$

where F^μ is four-force. Denoting

$$E_\Lambda \equiv -\int \Lambda_\rho \, d^3x \tag{22}$$

mentioned reasoning yields

$$H^\mu H_\mu - m^2c^2 = V^\mu V_\mu - 2m\gamma E_\Lambda \tag{23}$$

To ensure compliance with the equations of quantum mechanics it suffices that

$$\gamma E_\Lambda = \frac{V^\mu V_\mu}{2m} \tag{24}$$

By introducing quantum wave function Ψ

$$\Psi \equiv e^{\pm i K^\mu X_\mu} \tag{25}$$

where K^μ is wave four-vector related to cannonical four-momentum

$$\hbar K^\mu \equiv H^\mu \tag{26}$$

from (23) one obtains Klein-Gordon equation

$$\left(\Box + \frac{m^2 c^2}{\hbar^2} \right) \Psi = 0 \tag{27}$$

It seems, that considered method [2] may allow the analysis of the system in the quantum approach, classical approach and the introduction of a field-dependent metric for curved spacetime, which may help with connecting previously divergent descriptions of physical systems.

As it was shown in [2], there is also possibility to obtain Hamiltonian density that agrees with the classical Hamiltonian density for electromagnetic field, considered in Quantum Field Theory. Such Hamiltonian density was currently considered mainly for sourceless regions and to consider the system with electromagnetic field only.

According to the presented, unconfirmed yet results, it may appear that, actually, this Hamiltonian density describes the entire physical system, containing all known interactions. For this reason, the discussed method might also greatly simplify Quantum Field Theory equations.

REFERENCES

[1] P. Ogonowski, Proposed method of combining continuum mechanics with einstein field equations, International Journal of Modern Physics D **2350010**, 15 (2023).

[2] P. Ogonowski, Developed method. interactions and their quantum picture (2023), arXiv:2306.14906 [physics.gen-ph].