Backreaction induced modification in spacetime structure: current acceleration and gravitational wave propagation

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Our present Universe contains inhomogeneous matter distribution at considerably large scales. We evaluate the effect due to backreaction of the inhomogeneities on the structure of spacetime in the framework of Buchert's averaging procedure. Investigating the late time global evolution, we show that the Universe can transit from the presently accelerating phase to undergo future deceleration [1–3]. We constrain the parameters of our spacetime model consisting of inhomogeneous matter distribution in multiple domains through a Markov Chain Monte Carlo method using the Union 2.1 supernova Ia data [4]. Further, the redshift-distance relation is seen to get modified even in the context of a simplified two-domain model within the above framework. Considering the observed gravitational waves emitted by compact binaries, we show that the variation of the redshift dependent part the gravitational wave amplitude can deviate significantly compared to that in the ΛCDM-model [5]. Moreover, the presence of viscous components in the cosmic fluid can lead to additional impact on the gravitational wave observables [6]. Our results signify the importance of modified spacetime structure due to matter inhomogeneities on precision measurements of parameters of gravitational wave sources.

The standard cosmological (ΛCDM) model is based on the assumption of homogeneity and isotropy of the Universe at large scales, and is governed by the FLRW metric of spacetime. However, current observations [7] indicate that matter inhomogeneities extend beyond the of superclusters of galaxies even up to scales of $500h^{-1}Mpc$. Backreaction of inhomogeneities can have significant consequences on the spacetime structure and global evolution of the Universe [8, 9]. Here we employ Buchert's backreaction formalism [8, 10, 11] to investigate the late time global evolution of the Universe in the presence of observed matter inhomogeneities.

Gravitational waves act as complementary messengers to electromagnetic waves opening up a new window to the physics of the Universe. The recent excitement in the field stems from several detections of gravitational waves from compact binary mergers since the first report by the LIGO and VIRGO scientific collaborations [12]. The observed parameters of gravitational waves are significant for inferring parameters associated with the sources, such as their mass range and merger rates. Our goal here is to study the effect of local inhomogeneities in the Universe on the propagation of gravitational waves (GW).

We consider a model of the Universe in which the global domain D comprises of multiple subregions of two types - (i) overdense regions which are closed dust-only FLRW regions with a positive curvature and a deceleration parameter $q_o = -\ddot{a_o}/a_o H_o^2 > 0.5$, and (ii) underdense regions which are flat (zero intrinsic curvature) FLRW regions and having smaller density (as compared to the overdense regions). There are in total n subregions in our model, i number of them are overdense and $(n-i)$ of them are underdense. The scale factor and time of the i^{th} overdense region evolve with development angle ϕ of the overdense region as [3], $a_{o_i} = \frac{q_{o_i}}{2q_{o_i}-1}(1-\cos\phi)$, and $t_i = \frac{q_{o_i}}{2q_{o_i}-1}(\phi - \sin\phi)$, respectively, where q_{o_i} is the deceleration parameter of the i^{th} overdense region. The scale factor of the i^{th} under-dense region is taken to evolve as a function of time t given by $a_{u_i} = c_{u_i} t^{\beta_i}$. Here c_{u_i} and β_i are constants which determine the time evolution of the i^{th} under-dense subregion. β_i varies from $2/3$ to 1 to denote any behaviour ranging from a matter-dominated up to an accelerating region.

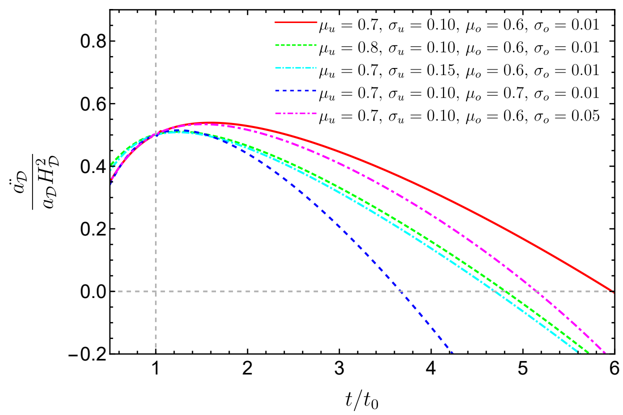

FIG. 1. Evolution of the global acceleration parameter $\frac{\ddot{a_\mathcal{D}}}{a_\mathcal{D} H_\mathcal{D}^2}$, for different sets of model parameters.

Now, applying the Bucherts' backreaction formalism,

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the expression of the global acceleration takes the form

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \left(\sum_i -\lambda_{o_i} q_{o_i} H_{o_i}^2 \right) + \left(\sum_j \lambda_{u_j} \frac{\beta(\beta-1)}{t^2} \right) + \left(\sum_k \sum_l \lambda_k \lambda_l \left(H_l - H_k \right)^2 \right). \tag{1}$$

Here λ_{o_i} and λ_{u_j} are the volume fractions of the i^{th} overdense and underdense regions, respectively. H_{o_i} is the Hubble parameter of the i^{th} overdense region, λ is the set of all λ_{o_i} and λ_{u_i} and H is the set of all H_{o_i} and H_{u_i}. The total volume fraction of all the under-dense regions, i.e. $\sum_i \lambda_{u_i}$ is given by λ_u, and similarly, for all the over-dense regions ($\lambda_o + \lambda_u = 1$). We assume Gaussian distributions for volume fractions of underdense $\lambda_{u_i,0}$ and over-dense sub-regions $\lambda_{o_i,0}$, given by [4],

$$\lambda_{u_i} = \frac{N_u}{\sigma_u \sqrt{2\pi}} e^{-(\beta_i - \mu_u)^2/2\sigma_u^2}, \quad \lambda_{o_i} = \frac{N_o}{\sigma_o \sqrt{2\pi}} e^{-(q_{o_i} - \mu_o)^2/2\sigma_o^2} \tag{2}$$

In Fig. 1, the acceleration of the Universe $\frac{\ddot{a_{\mathcal{D}}}}{a_{\mathcal{D}} H_{\mathcal{D}}^2}$ is plotted for different sets of model parameters *viz.*, μ_o, σ_o, μ_u and σ_o. Here, one hundred under-dense and one hundred over-dense sub-domains are taken into account. We fix the present value (at $t = t_0$) of the global acceleration parameter $\frac{\ddot{a_{\mathcal{D}}}}{a_{\mathcal{D}} H_{\mathcal{D}}^2}$ to be 0.55 obtained from Planck 2018 results [13]. It can be seen that the acceleration parameter begins to fall off beyond the present era ($t = t_0$), and the Universe transits to a decelerating phase ($\frac{\ddot{a_{\mathcal{D}}}}{a_{\mathcal{D}} H_{\mathcal{D}}^2} < 0$ at a subsequent time. For higher values of the model parameters, namely, μ_o, σ_o, μ_u or σ_o, the acceleration parameter $\frac{\ddot{a_{\mathcal{D}}}}{a_{\mathcal{D}} H_{\mathcal{D}}^2}$ falls more rapidly with time.

Let us now consider a 2-domain approximation of our spacetime model in terms of one underdense and one overdense region [1–3, 5, 6, 10]. Such a model can be analogously depicted in terms of a scalar field model ensuring observational constraints on the "morphon" potential [3]. The red-shift versus distance relation undergoes interesting modifications in context of Buchert backreaction formalism applied to such two-scale models [5, 14]. Here we study the amplitude of GW emitted from compact binaries, and traversing through a region of the Universe with inhomogeneous matter distribution. The red-shift dependent part of the GW amplitude in the presence of viscous inhomogeneities is given by [6]

$$F(z) = \frac{(1+z)^{5/3}}{D_L} e^{-\frac{\eta}{2(1+z)^2} \frac{D_L}{\left(1+\left(\frac{a_u}{a_o}\right)^3\right)}} \tag{3}$$

with D_L being the luminosity distance of the GW source. In (Fig. 2), we plot the redshift dependent part of the GW amplitude, for the ΛCDM model as well as for our backreaction model with or without viscosity. Since the evolution of the scale factor is altered by bulk viscous terms in the cosmic fluid the observed GW amplitude is affected by both types of (bulk and shear (η)) viscosities.

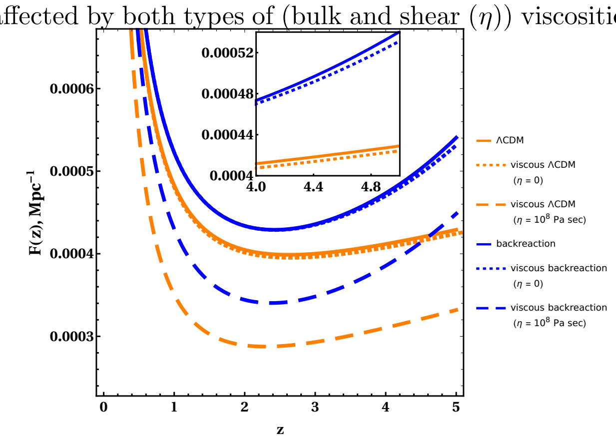

FIG. 2. Plot of GW amplitude versus redshift for various models. Inset shows the magnified portion of the solid (non-viscous) and the broken (viscous) curves.

To summarize, the presence of inhomogeneity in the present matter distribution of the Universe at considerably large scales can alter the late time global evolution through backreaction effects. The current phase of acceleration may switch over to an epoch of future deceleration. The altered structure of spacetime leads to modifications in the gravitational wave observables. Consideration of such modified GW observables is significant for estimation of source parameters of compact binaries in the present era of precision cosmology.

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