Torsion and Chern-Simons gravity in 4D space-times from a Geometrodynamical four-form

Patrick Das Gupta

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Department of Physics and Astrophysics, University of Delhi, Delhi - 110 007

Abstract

By envisaging a 4D space-time manifold described by a line-element $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu$, Hermann Minkowski had overturned the idea of viewing space and time disjointly. It is interesting to note that in the flat Minkowski space-time not only the Minkowski metric $\eta_{\mu\nu}$ is invariant under proper Lorentz transformations, the totally antisymmetric Levi-Civita tensor is invariant too. Now in Einstein's general relativity, $\eta_{\mu\nu} \to g_{\mu\nu}(x)$ in curved space-times with $g_{\mu\nu}$, the space-time dependent metric tensor, becoming dynamical. Analogously, in the present study, it is postulated that the Levi-Civita tensor $e_{\mu\nu\alpha\beta} \to w_{\mu\nu\alpha\beta} \equiv \phi(x)e_{\mu\nu\alpha\beta}$ takes a dynamical role. It is shown that this geometrodynamical four-form field, $\tilde{w} = \frac{1}{4!}w_{\mu\nu\rho\sigma}\tilde{dx}^\mu \wedge \tilde{dx}^\nu \wedge \tilde{dx}^\rho \wedge \tilde{dx}^\sigma$, augments general relativity by leading naturally to torsion as well as a Chern-Simons extension of the theory. Also, it can be demonstrated that the unique scalar-density $\phi(x)$ associated with $\tilde{w}$ can be used to construct a generalized exterior derivative that converts a p-form density to a (p+1)-form density of identical weight.

In order to put observational constraints on the four-form field $\tilde{w}$, we first note that the associated scalar-density $\phi(x)$ corresponds to a pseudo-scalar field too (like the axion) in the Minkowski space-time, and we had previously shown that such an ultra-light pseudo-scalar perhaps is the prime candidate for dark matter (DM). The quantum evolution of a self-gravitating Bose-Einstein condensate (BEC) of such ultra-light dark bosons can naturally give rise to the formation of supermassive black holes (SMBHs) on time scales of only $\sim 10^8$ years, thus settling a nagging problem concerning the frequent discovery of several billion solar mass heavy SMBHs at epochs when the universe was barely a billion years old. Furthermore, perturbations in the DM BEC induced by the tidal forces of interacting galaxies can not only cause bursts of star formation in the galactic nuclei but also result in nano-Hertz gravitational wave (GW) generation that may be constrained using Pulsar Timing Arrays.

References:

P.D. G. and Thareja, E., 2017. Supermassive black holes from collapsing dark matter Bose–Einstein condensates. Classical and Quantum Gravity, 34(3), p.035006.

P.D.G., 2010. On Chern–Simons corrections to magnetohydrodynamics equations. Radiation Effects & Defects in Solids: Incorporating Plasma Science & Plasma Technology, 165(2), pp.106-113.

P.D.G. and Rahman, F., 2018. Aspects of Black Hole Physics and Formation of Super-massive Black Holes from Ultra-light Dark Bosons. The Physical Universe, eds. S. M. Wagh, S. D. Maharaj & G. Chon (2018, Published by Central India Research Institute, India) arXiv preprint arXiv:1801.02559.

P.D.G., 2009. Dark energy and Chern-Simons like gravity from a dynamical four-form. arXiv preprint arXiv:0905.1621.