Inertial Structure Without Elimination: Three Objections to Barbour-Bertotti Intrinsic Dynamics

Barbour and Bertotti's 1982 program of intrinsic dynamics proposes to eliminate inertial frames and absolute time from the foundations of classical mechanics by reformulating dynamics on a relational configuration space Q_0, invariant under the Leibniz group. We argue that this program, while technically ingenious, fails on three grounds: it misdiagnoses the role of inertial frames in Newtonian theory, cannot escape the operational need for inertial structure in writing equations of motion, and covertly reintroduces temporal structure through the very metric it requires for best-matching.

First, Barbour's central motivation — that Newtonian mechanics ontologically presupposes inertial frames — misreads the epistemic status of those frames. Inertial frames are not primitive posits but empirical findings: an observer determines whether they occupy an inertial frame by checking whether the accelerations they observe, multiplied by the relevant masses, match the independently identified force sources. If they do, the frame is inertial; if they do not, it is not. This is not a presupposition of inertial structure but a procedure for discovering it. The concept of inertial frame is therefore constitutively defined by Newton's laws rather than presupposed by them, as DiSalle has argued. Barbour's relational program is thus motivated by a problem that does not exist in the form he diagnoses.

Second, even granting the relational framework, recovering the equations of motion from intrinsic dynamics requires knowing the potential $V(q)$ and the rotational state of one's frame relative to inertial structure. In a rotating frame, predicting the dynamics of a physical system requires knowing Ω — the frame's rotation rate — which itself presupposes that the frame has been identified as rotating relative to something with inertial significance. This is not merely a practical difficulty but a structural one: the recovered Newtonian equations cannot be applied to make predictions without information about the frame's relationship to inertial structure. One cannot bootstrap physical predictions purely from within a frame-agnostic description. General relativity illuminates why. Even in non-inertial frames, GR does not achieve frame-agnosticism by eliminating the inertial-non-inertial distinction; rather, it achieves coordinate independence by systematically decomposing every measurement into a gauge-invariant part — proper acceleration, measured by an accelerometer and reflecting genuine physical contact forces — and a gauge-variant part arising from coordinate choice, encoded in the Christoffel symbols. The geodesic equation $\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = a^\mu$makes this decomposition explicit. Physics is not frame-agnostic; it is the disciplined separation of physical from conventional content, and that separation is neither arbitrary nor eliminable.

Third, the best-matching procedure at the heart of intrinsic dynamics covertly presupposes the temporal structure it claims to eliminate. To define the intrinsic distance between two successive configurations in Q_0, Barbour must compare lengths and distances across time — he must treat certain geometrical relationships as stable from one configuration to the next. But this is precisely a commitment to rigidity across time: a definition of which physical quantities remain constant as the Universe evolves. This is not an arbitrary choice — it is constrained by minimizing change — but it is nonetheless a temporal commitment. The metric on configuration

space that best-matching requires cannot be defined without already presupposing that spatial distances are comparable across successive instants. Far from eliminating time, intrinsic dynamics encodes it in the structure of Q_0 itself. Barbour's claim to have dispensed with independent time therefore succeeds only formally: the label time λ is removed from the action, but temporal structure re-enters through the rigidity assumptions built into the configuration space metric that makes best-matching well-defined in the first place.

Together these objections suggest that inertial structure and temporal comparability are not optional metaphysical excesses to be engineered away, but indispensable conditions for the empirical content of dynamical theory. The Machian program illuminates important features of the relational structure of mechanics, like reparameterization invariance and variational principle, but it does not succeed in eliminating the structures it targets.