Material Media and Vacuum as Operational Solution Classes of a Spacetime Transport Cauchy Problem: Classification, Physical Realization, and Diagnostics

Georgios N. Kallidis, Nikolaos G. Kallidis

Nicosia, Cyprus · *geokallides@gmail.com*

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This paper demonstrates that material media and the vacuum are not independent ontological primitives but mutually exclusive solution classes of a single spacetime transport Cauchy problem in 1+1 dimensions. Starting from finite, SI-traceable measurement protocols—clock synchronisation, bidirectional equidistance, ruler transport, and clock-rate comparison—the construction generates a smooth operational manifold and coupled first-order transport equations without presupposing any background metric, Lorentzian causal structure, or one-way light-speed convention. The extended Poincaré transformations, velocity composition, and the Minkowski-type invariant emerge as derived outputs. This abstract highlights results from Parts II–IV: the metrological interval equivalence, the material–vacuum bifurcation, the physical realization of the mixed matter–vacuum sector including the Sandage time-dilation theorem, and the finite-data operational diagnostics.

Metrological interval equivalence (Part II). A key preparatory result, established in § 14 of the paper before any material–vacuum classification is introduced, concerns the relationship between electromagnetic wave parameters and spacetime intervals. It is shown that the period and wavelength of electromagnetic waves exchanged between two particles transform under inter-frame passage by exactly the same Doppler factors as the corresponding time and space intervals between their emission and reception events. The wave period T and the propagation time interval $\Delta\tau$ both scale as $\sqrt{(1\mp v/c)/(1\pm v/c)}$; the wavelength λ and the propagation spatial interval $\Delta\xi$ transform identically. Consequently, an exchanged electromagnetic wave can be interpreted either as a counted sequence of cycles or as a spacetime interval resolved by the transport kinematics. This equivalence is not a convention but a theorem of the operational transport structure. Its significance becomes apparent in Part IV, where it provides the bridge between microscopic frequency redshift and macroscopic time dilation.

Bifurcation theorem (Part III). The operational manifold M is constructed from inertial-frame affine charts with extended Poincaré transformations as transition maps, yielding a Lie group $M \cong (\mathbb{R}^2, +)$ whose atlas transition group is $\mathrm{Poin}(1,1) = \mathbb{R}^2 \rtimes O(1,1)$. The temporal and spatial transport fields $X = \partial_t + v\,\partial_x$ and $T = \partial_x + (v/c^2)\,\partial_t$ span a rank-two distribution $D = \mathrm{span}\{X, T\}$. The central result is that the homogeneous–isotropic (HI), protocol-closed transport problem admits *exactly two* pure solution classes, distinguished by their infinitesimal symmetry algebras on D:

- **Material branch:** admissible generators preserve the rest decomposition $D = \mathrm{span}\{X\} \oplus \mathrm{span}\{T\}$, so $[X, T] = 0$ and the Frobenius theorem yields a comoving rest frame. This branch carries a rest-preserving Lie algebra $\mathfrak{g}_{\mathrm{rest}}$.
- **Vacuum branch:** the boost generator $U = -t\,\partial_x - (x/c^2)\,\partial_t$ mixes the transport directions ($[U, X] = T$, $[U, T] = (1/c^2)X$), admitting no invariant rest line. The sole invariant structures are the optical eigenlines $N_\pm = T \pm (1/c)\,X$, with $[U, N_\pm] = \pm(1/c)\,N_\pm$. This branch carries an optical Lie algebra $\mathfrak{g}_{\mathrm{opt}}$.

These two classes are proved mutually exclusive and exhaustive. The distinction is a structural bifurcation at the Lie-algebra level, not a pointwise kinematic classification.

Physical realization of the mixed sector (Part IV). On the vacuum branch, the optical eigenlines determine null propagation at speed c, yielding the d'Alembertian: $N_+N_-\Phi = 0 \Leftrightarrow \partial_t^2\Phi - c^2\partial_x^2\Phi = 0$. On the material branch, the commuting transport fields generate continuum balance laws for energy $E = \gamma mc^2$ and momentum $P = \gamma mv$. The *inter-branch characteristic correspondence* shows that source-free material energy–momentum modes diagonalise into $w_\pm = E \pm cP$ satisfying $\partial_t w_\pm \pm c\,\partial_x w_\pm = 0$—the same null transport equations as the vacuum optical modes.

The mixed matter–vacuum sector, generated by $Z = aY + bU$, defines an effective refractive index $n_{\mathrm{eff}} = \gamma^2 b/(ac)$ and propagation speed $c_{\mathrm{eff}} = c/n_{\mathrm{eff}} < c$ for excess coupling $\chi := n_{\mathrm{eff}} - 1 > 0$. This single transport law yields three physical realisations: (i) discontinuous spatial coupling → characteristic refraction (Snell's law); (ii) static spatial coupling $\chi_G(r) = \kappa GM/(c^2 r)$ → Shapiro-type delay; (iii) homogeneous temporal coupling $\chi(\tau)$ → cosmological redshift: $1 + z = n_{\mathrm{eff}}(\tau_r)/n_{\mathrm{eff}}(\tau_e)$.

Sandage time-dilation theorem (Part IV). The cosmological redshift realisation, combined with the

Part II metrological interval equivalence, yields a further structural result. Since electromagnetic wave periods and light-propagation time intervals realise the same operational transport ratio, the framework predicts that the macroscopic duration of any extended optical signal scales with exactly the same factor as the microscopic frequency redshift: $\Delta\tau_r/\Delta\tau_e \simeq n_{\text{eff}}(\tau_r)/n_{\text{eff}}(\tau_e) = 1 + z$. This is proved as a theorem of the operational transport structure: microscopic wavelength stretching and macroscopic time dilation are two projections of one metrological transport law, not two independent cosmological coincidences. Crucially, the interval equivalence on which the theorem rests was derived in Part II from the operational reconstruction itself, independently of any cosmological context; the Sandage scaling emerges only afterward, when the same transport law is applied to a globally homogeneous, time-dependent mixed coupling $n_{\text{eff}}(\tau) = 1 + \chi(\tau)$. The result therefore reproduces the observational signature that historically ruled out purely dissipative tired-light models—without invoking metric expansion as the foundational mechanism.

Operational diagnostics (Part IV). A three-stage certification scheme translates the algebraic classification into finite-data tests. *Stage I* certifies HI closure via advection residuals ρ_1, ρ_2 and synchronisation anisotropy α. *Stage II* certifies materiality through rest-preserving commutator closure. *Stage III* certifies vacuumity through boost-mixing and optical-eigenline preservation. These diagnostics are stable under passage between forward and comoving frames.

Significance for spacetime ontology. The framework addresses questions pertinent to this conference. First, *causality is established pre-metrically*: the null characteristic structure precedes any metric or invariant interval, offering a causal architecture that does not presuppose the geometry it constrains. Second, the *vacuum is an operationally certified medium*, separated from the material branch by a theorem and certified through finite measurements. Third, the Sandage theorem carries a structural cosmological observation: backward extrapolation of positive redshift yields $\chi \to 0$, $n_{\text{eff}} \to 1$, $c_{\text{eff}} \to c$—a regular pure-vacuum transport limit rather than a singular metric boundary. Metric expansion, on this reading, may represent an effective geometric encoding of a deeper metrological matter–vacuum transport dynamics. The dynamical law governing the evolution of χ remains an open problem for subsequent work.