Foundations of Torsion in Theories of Gravity

1 Topic Proposal

Among the various modifications of the geometry of space-time, torsion arguably represents the smallest structural departure, since it merely quantifies the asymmetry of the connection coefficients of the affine structure (Hehl et al. 1976). Such modification is mainly motivated by the trust in general relativity, and by the formulation of gravity as a gauge theory (Hammond 1995). A manifold endowed with a metric compatible with such an affine structure is usually referred to as an Einstein–Cartan space-time, in which torsion is generally sourced by spin density (see, however, Schrödinger 1943) in a way analogous to how energy–momentum sources the gravitational field (Hammond 2002; Shapiro 2002). Such space-time admits a geometric and gravitational interpretation similar to that of Einstein's theory (Lehmkuhl 2008). However, the conceptual simplicity with which the geometry of general relativity is usually understood – for instance, comprising an affine structure compatible with the metric whose geodesics are followed by free particles – is completely lost in theories with torsion. What is surprising is not merely the fact that autoparallel and extremal curves no longer coincide, but rather that particles are expected to follow different trajectories which, in most cases, are neither autoparallel nor extremal (Yasskin and Stoeger 1980). If correct (and it might not be so), this feature would seriously undermine the Machian framework within which Einstein developed general relativity, according to which there is an action–reaction relation between matter and space-time, whereby the latter determines the space-time paths of free particles (Brown 2005). With torsion, it appears that space-time does not "force" particles to follow its geodesics anymore.

In this talk, I investigate the foundations of theories with torsion by addressing the question of how torsion should be interpreted in the first place. This question remains unclear throughout much of the physics literature on modified theories of gravity. On the one hand, torsion admits a well-defined geometrical characterization, being generally associated with a generalized affine structure and with the failure of closure of infinitesimal parallelograms. This characterization, however, is only partially accurate: in fact it could be argued that the fundamental (geometrical) effect of torsion on vectors is only associated with an additional first-order rotation in the covariant derivative. On the other hand, torsion is often treated as an independent matter field, which may be related to spin density either algebraically (as in Cartan's original formulation) or dynamically (Hammond 1995; Stoeger 1985). This is what seems to explain the deviation from geodesic motion in the first place, thereby contradicting the way the affine structure of Einstein-Cartan is introduced. From this point of view, the framework in which torsion is explained might represent a further case for the space-time/matter dichotomy discussed, for instance, by Martens and Lehmkuhl (2020b), Martens and Lehmkuhl (2020a), and Ferreiro, Fleuren, and Martens (2025), since it seems not to have a well-defined ontological categorization, by having both geometric and matter-like aspects. Moreover, a space-time endowed with torsion raises doubts about which criteria are required for an entity to be regarded as space-time – given that both the weak geodesic and the chronogeometricity criteria would cease to apply (Martens and Lehmkuhl 2020a) – and even about which structure should be the one followed by particles.

Motivated by these open questions, several foundational aspects of general relativity are called into question with torsion theories of gravity: for example, it is no longer straightforward to

determine which stress–energy tensor should be regarded as fundamental – namely, the canonical or the metric one. Stress-energy is a fundamental tool for determining the particles' paths (Geroch and Weatherall 2018): interestingly enough, the covariant derivative of neither of them vanishes. Furthermore, it is not straightforward to define how the equivalence principles ought to be extended to the Einstein–Cartan space-time (Di Casola, Liberati, and Sonego 2015; Lehmkuhl 2008), thereby opening the question as to which geometric quantity – i.e. the metric or the connection coefficients – should be regarded as the more fundamental from a geometrical point of view.

Therefore, this work is of great relevance for the debate about the metaphysics of space-time: it comprises relevant questions such as the geometric structure that is necessary to represent space-time, the relation, distinction and action-reaction between space-time and matter, the unspecified assumptions hidden in gravitational theories associated with geodesic motion and, finally, the role of the equivalence principles in gravity.

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