PIEs with ERBs in

Abstract

The unification of space and time via Lorentzian signature was a paradigm shift in our understanding of the Universe. QM however resists the inclusion of a time operator due to the Pauli theorem, inspiring a variety of explanations for time's emergence on large scales. This work considers an alternative resolution - that rather than time disappearing on small scales, it's apparent timelike nature may arise from the structure of spacetime, with the fundamental nature of time instead being spacelike.

The fact that a particle at rest propagates through time at the same speed as a massless field propagates through space is certainly suggestive of a commonality of form between space and time. So how might a Lorentzian metric be constructed from only spacelike dimensions and what would the implications of such a construction be? This work is directed at addressing these questions, albeit in a restricted sense, by replicating the form of the Schwarzchild metric using extended spatial dimensions only.

Lorentzian transformations are a combination of Galilean and Carrollian transformations, plus a length contraction, time dilation and spatial rotation [1]. Inspired by this, the two sheets of an Einstein-Rosen bridge (ERB) are taken to be effectively a Galilean and a Carrollian spacetime. Galilean and Carrollian transformations are also related by the exchange of space and time [1] and here too the temporal sector of each spacetime is normalised by the same function as the other's radial direction, such that the roles of space and time are interchanged between the two.

This is the crux of the construction and involves for example, the square root of the contravariant form of the radial direction's normalisation from the Carrollian sheet appearing in the (covariant) clock one-form of the Galilean sheet, as can be seen from there generalised definitions,

$$\text{Galilean:} \quad u_\mu = \left(\tfrac{1}{f(\tilde{r})}, 0, 0, 0\right), \quad h^{\mu\nu} = \left(0, \tfrac{1}{f(r)^2}, \tfrac{1}{r^2}, \tfrac{1}{r^2 \sin^2\theta}\right) \tag{1}$$

$$\text{Carrollian:} \quad v^\mu = (f(r), 0, 0, 0), \quad \tilde{h}_{\mu\nu} = \left(0, f(\tilde{r})^2, \tilde{r}^2, \tilde{r}^2 \sin^2\theta\right) \tag{2}$$

The Galilean spacetime is now the $c \to \infty$ limit of the Lorentzian metric,

$$ds^2 = -f(\tilde{r})^{-1} \mathrm{d}\tilde{r}^2 + f(r) dr^2 + r^2 d\Omega^2, \tag{3}$$

whilst the Carrollian spacetime is the $c \to 0$ limit of the metric,

$$ds^2 = -f(r)^{-1} \mathrm{d}r^2 + f(\tilde{r}) d\tilde{r}^2 + \tilde{r}^2 d\Omega^2. \tag{4}$$

with both having the same form as Schwarzchild's. These metrics are equivalent under the exchange $r \leftrightarrow \tilde{r}$, so that the volume form, which is a psuedo-vector, flips sign from one side of the bridge to the other. This reverses the overall sign of the Einstein-Hilbert action, thus admitting an interpretation involving negative relative energies between the two sheets. In addition however, this is equivalent to flipping the overall signature of the metric, (parametrised by $\sigma = \pm 1$ in Eq. (5)) along with a simultaneous change in the sign of the cosmological constant [2],

$$S_{\text{EH}} \;=\; \frac{1}{16\pi G} \int \mathrm{d}^4x \, \sqrt{|g|} \, (\sigma R - 2\Lambda). \tag{5}$$

The Schwarzchild metric constructed in this way is reminiscent of the CPT symmetric 'black mirror' solution of [3]. Here, the exchange of space and time would then be a fourth symmetry, arising from the relationship between Galilean and Carrollian transformations and their role as constituent parts of Lorentzian transformations.

The identification of the Carrollian sheet's radial direction with proper time τ meanwhile, can be seen to provide a natural description of the spacetime around a particle interaction event (PIE) upon which a massive particle at rest can be represented as a massless field. The Galilean sheet can be identified with the asymptotically de-Sitter free space we see around us (itself appearing very close to Galilean), whilst the proper-time 3-space would be seen as an asymptotically anti-de-Sitter space internal to particle interaction events, themselves identified with black-holes which immediately evaporate via Hawking radiation.

Some of the implications for particle physics and quantum mechanics that such a two-sheeted spacetime would have are then considered. The Pauli theorem need not necessarily apply for example, since the two sheets can be described as having energies of opposing signs. Proper time with mass as its conjugate can then be elevated to the status of quantum mechanical operators and find a natural home in the exponential factor of particle wavefunctions $\exp[-\frac{i}{\hbar}(Et/c - pr - mc\tau)]$.

In this framework, the first derivatives of a wavefunction provide only local energies for a given sheet, i.e. either $E = pc$ or mc^2. The usual relativistic energy constraint is given by a simplified Klein-Gordon equation $\Box\phi = 0$, which becomes precisely the square of a simplified Dirac equation $\gamma^\mu \partial_\mu \psi = 0$, without the usual need to choose opposite signs for the mass term by hand. The Proca equation also takes the more compact form $\partial_\mu \partial_\mu B^\nu = 0$ and the Schrödinger equation (up to a sign) arises straightforwardly from the application of the Laplacian operator $\Delta\psi = \nabla^2\psi$ and taking the non-relativistic limit $E \approx mc^2$.

It is also noted that associating the complex conjugate of a particle's wavefunction with its aspect on a second sheet would make mutually exclusive states on the Bloch sphere invariant under Lorentz boosts, eliminate the need to use γ^0 in the definition of dual spinors and allow the Klein-Gordon equation to return positive definite probability densities.

References

[1] N. D. Sen Gupta. On an analogue of the Galilei group. *Il Nuovo Cimento A (1965-1970)*, 44(2):512–517, July 1966. ISSN 1826-9869. doi: 10.1007/BF02740871. URL `https://doi.org/10.1007/BF02740871`.

[2] Bruno Alexandre, Steffen Gielen, and João Magueijo. Overall signature of the metric and the cosmological constant. *JCAP*, 02:036, 2024. doi: 10.1088/1475-7516/2024/02/036.

[3] Kostas Tzanavaris, Latham Boyle, and Neil Turok. Black mirrors: Cpt-symmetric alternatives to black holes, 2025. URL `https://arxiv.org/abs/2412.09558`.