Minkowski's actual and intended contributions to spacetime physics

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My talk is dedicated to the 160th anniversary of the birth of Hermann Minkowski (1864-2024). He was born on June 22, 1864 and departed from this strange world on January 12, 1909 at the age of 44.

This year, when we mark Minkowski's anniversary, it is natural to recall and re-examine his epoch-making contributions to spacetime physics. Given the depth of Minkowski's thinking and the far-reaching consequences of his results, it is also natural to wonder how fundamental (at least spacetime) physics might look like now if he had lived longer. In this sense, it is worth trying to imagine what Minkowski's intended[1] contributions might have been.

In the first part I will discuss Minkowski's major and actual (not the widely shared) contribution – the discovery of the spacetime structure of the world, i.e., the discovery of spacetime physics itself. In his Foreword to the second edition of the collection of papers *The Origin of Spacetime Physics* [1] Ashtekar nicely describes the application of the developed by Minkowski four-dimensional mathematical formalism of spacetime physics to electrodynamics:

> The new edition includes a Chapter based on Minkowski's lecture to the Göttingen Scientific Society on December 21st, 1907, entitled *Fundamental Equations for the Electromagnetic Processes in Moving Bodies*. This is a much more detailed account of Minkowski's astonishingly deep understanding of how the fusion of space and time into a four-dimensional spacetime continuum leads to a reformulation of electrodynamics. In particular, this paper provides the tensorial formulation of Maxwell's equations and the action of the Lorentz group on the Maxwell field tensor and the source current. Because of its emphasis on four-dimensional geometry, this discussion of Maxwell's equations goes distinctly beyond Einstein's paper on *On the Electrodynamics of Moving Bodies*. Indeed, Minkowski's four-dimensional equations are exactly in the same form that we use today, more than a century later!

[1] I think the word "intended" is the appropriate one in view of Minkowski's program of geometrizing physics, which he intended to pursue.

After that I will summarize the evidence that Minkowski arrived independently of Einstein at the equivalence of the times of observers in relative motion and independently of Poincaré at the conclusion that the Lorentz transformations imply a four-dimensional space, but Einstein and Poincaré published first while Minkowski had been developing the full-blown four-dimensional formalism of spacetime physics; he did not publish his results earlier "because he wished first to work out the mathematical structure in all its splendour" (M. Born [2]).

In the second part of the talk I will describe what Minkowski's intended contributions to spacetime physics might have been. What enables me to talk (not just speculate) about what Minkowski might have achieved is his explicitly outlined program of geometrizing physics – regarding four-dimensional physics merely as geometry of spacetime, where material objects are a forever-given (static) web of worldlines [3]:

> The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.

Particularly, I will discuss how applying his program of geometrizing physics Minkowski (had he lived longer) might have arrived before Einstein (or independently of him) at the idea that gravitation is a manifestation of the non-Euclidean geometry of spacetime.

References

[1] V. Petkov (ed.), *The Origin of Spacetime Physics*, 2nd ed., with a Foreword by A. Ashtekar (Minkowski Institute Press, Montreal 2023).

[2] M. Born, *My Life: Recollections of a Nobel Laureate* (Scribner, New York 1978) p. 131.

[3] H. Minkowski, Space and Time, in [1, p. 148].